

CP Violation in B Decays

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Abstract

In the near future, we will have the first significant experimental measurements of CP violation in B decays. These measurements will easily test crucial questions such as whether the Standard Model Kobayashi-Maskawa phase plays a dominant role in CP violation or whether CP is an approximate symmetry in nature. We explain the different types of CP violation in B decays, and the usefulness of measuring them. We use the same formalism to describe the ε_K and ε'_K parameters of the neutral K system and to explain the terms direct and indirect CP violation. We present the Standard Model predictions for the various asymmetries. We argue that certain CP asymmetries in B decays are subject to a very clean theoretical interpretation in terms of fundamental Lagrangian parameters. Within the Standard Model, these asymmetries will provide very accurate measurements of the CKM parameters. In case that deviations from the Standard Model predictions will be found, there is enough information to understand the nature of New Physics that is required to explain them. We demonstrate this statement by analyzing the impact of various Supersymmetric flavor models on CP violation.

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I. A MODEL INDEPENDENT DISCUSSION

A. Introduction

CP violation arises naturally in the three generation Standard Model. The CP violation that has been measured in neutral K -meson decays (ε_K and ε'_K) is accommodated in the Standard Model in simple way [1]. Yet, CP violation is one of the least tested aspects of the Standard Model. The value of the ε_K parameter [2] as well as bounds on other CP violating parameters (most noticeably, the electric dipole moments of the neutron, d_N , and of the electron, d_e) can be accounted for in models where CP violation has features that are very different from the Standard Model ones.

It is unlikely that the Standard Model provides the complete description of CP violation in nature. First, it is quite clear that there exists New Physics beyond the Standard Model. Almost any extension of the Standard Model has additional sources of CP violating effects. In addition there is a great puzzle in cosmology that relates to CP violation, and that is the baryon asymmetry of the universe [3]. Theories that explain the observed asymmetry must include new sources of CP violation [4]: the Standard Model cannot generate a large enough matter-antimatter imbalance to produce the baryon number to entropy ratio observed in the universe today [5–7].

In the near future, significant new information on CP violation will be provided by various experiments. The main source of information will be measurements of CP violation in various B decays, particularly neutral B decays into final CP eigenstates [8–10]. Another piece of valuable information might come from a measurement of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay [11–14]. For the first time, the pattern of CP violation that is predicted by the Standard Model will be tested. Basic questions such as whether CP is an approximate symmetry in nature will be answered.

It could be that the scale where new CP violating sources appear is too high above the Standard Model scale (*e.g.* the GUT scale) to give any observable deviations from

the Standard Model predictions. In such a case, the outcome of the experiments will be a (frustratingly) successful test of the Standard Model and a significant improvement in our knowledge of the CKM matrix.

A much more interesting situation will arise if the new sources of CP violation appear at a scale that is not too high above the electroweak scale. Then they might be discovered in the forthcoming experiments. Once enough independent observations of CP violating effects are made, we will find that there is no single choice of CKM parameters that is consistent with all measurements. There may even be enough information in the pattern of the inconsistencies to tell us something about the nature of the new physics contributions [15–18].

The aim of this review is to explain the theoretical tools with which we will analyze new information about CP violation. In this chapter, we give a brief, model-independent discussion of CP violating observables. In the next chapter, we discuss CP violation in the Standard Model. In the third chapter we briefly explain why CP violation is a powerful probe of new physics. In the last chapter, we describe CP violation in Supersymmetric models. This discussion enables us to elucidate the uniqueness of the Standard Model description of CP violation and how little it has been tested so far. It further demonstrates how the information from CP violation can help us probe in detail models of New Physics.

B. Neutral Meson Mixing

Much of the exciting CP violation in meson decays is related to neutral meson mixing. Before we focus on CP violation, we briefly discuss then the physics and formalism of neutral meson mixing. We refer specifically to the neutral B meson system, but most of our discussion applies equally well to the neutral K , B_s and D meson systems.

Our phase convention for the CP transformation law of the neutral B mesons is defined by

$$\text{CP}|B^0\rangle = \omega_B|\bar{B}^0\rangle, \quad \text{CP}|\bar{B}^0\rangle = \omega_B^*|B^0\rangle, \quad (|\omega_B| = 1). \quad (1.1)$$

Physical observables do not depend on the phase factor ω_B . An arbitrary linear combination of the neutral B -meson flavor eigenstates,

$$a|B^0\rangle + b|\bar{B}^0\rangle, \quad (1.2)$$

is governed by a time-dependent Schrödinger equation,

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a \\ b \end{pmatrix}, \quad (1.3)$$

for which M and Γ are 2×2 Hermitian matrices.

The off-diagonal terms in these matrices, M_{12} and Γ_{12} , are particularly important in the discussion of mixing and CP violation. M_{12} is the dispersive part of the transition amplitude from B^0 to \bar{B}^0 . In the Standard Model it arises only at order g^4 . In the language of quark diagrams, the leading contribution is from box diagrams. At sufficiently high loop momentum, $k \gg \Lambda_{\text{QCD}}$, these diagrams are a very good approximation to the Standard Model contribution to M_{12} . This, or any other contribution from heavy intermediate states from new physics, is the *short distance* contribution. For small loop momenta, $k \lesssim 1$ GeV, we do not expect quark hadron duality to hold. The box diagram is a poor approximation to the contribution from light intermediate states, namely to *long distance* contributions. Fortunately, in the B and B_s systems, the long distance contributions are expected to be negligible. (This is not the case for K and D mesons. Consequently, it is difficult to extract useful information from the measurement of Δm_K and from the bound on Δm_D .) Γ_{12} is the absorptive part of the transition amplitude. Since the cut of a diagram always involves on-shell particles and thus long distance physics, the cut of the quark box diagram is a poor approximation to Γ_{12} . However, it does correctly give the suppression from small electroweak parameters such as the weak coupling. In other words, though the hadronic uncertainties are large and could change the result by order 50%, the cut in the box diagram is expected to give a reasonable order of magnitude estimate of Γ_{12} . (For $\Gamma_{12}(B_s)$ it has been shown that local quark-hadron duality holds exactly in the simultaneous limit of small velocity and large number of colors. We thus expect an uncertainty of $\mathcal{O}(1/N_C) \sim 30\%$ [19,20]. For $\Gamma_{12}(B_d)$

the small velocity limit is not as good an approximation but an uncertainty of order 50% still seems a reasonable estimate.) New physics is not expected to affect Γ_{12} significantly because it usually takes place at a high energy scale and is relevant to the short distance part only.

The light B_L and heavy B_H mass eigenstates are given by

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle. \quad (1.4)$$

The complex coefficients q and p obey the normalization condition $|q|^2 + |p|^2 = 1$. Note that $\arg(q/p^*)$ is just an overall common phase for $|B_L\rangle$ and $|B_H\rangle$ and has no physical significance. The mass difference and the width difference between the physical states are given by

$$\Delta m \equiv M_H - M_L, \quad \Delta \Gamma \equiv \Gamma_H - \Gamma_L. \quad (1.5)$$

Solving the eigenvalue equation gives

$$(\Delta m)^2 - \frac{1}{4}(\Delta \Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \quad \Delta m \Delta \Gamma = 4\mathcal{R}e(M_{12}\Gamma_{12}^*), \quad (1.6)$$

$$\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta \Gamma} = -\frac{\Delta m - \frac{i}{2}\Delta \Gamma}{2M_{12} - i\Gamma_{12}}. \quad (1.7)$$

In the B system, $|\Gamma_{12}| \ll |M_{12}|$ (see discussion below), and then, to leading order in $|\Gamma_{12}/M_{12}|$, (1.6) and (1.7) can be written as

$$\Delta m_B = 2|M_{12}|, \quad \Delta \Gamma_B = 2\mathcal{R}e(M_{12}\Gamma_{12}^*)/|M_{12}|, \quad (1.8)$$

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}. \quad (1.9)$$

C. CP Violation in Neutral Meson Mixing

To discuss CP violation in mixing (see below), it is useful to write (1.9) to first order in $|\Gamma_{12}/M_{12}|$:

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} \left[1 - \frac{1}{2} \text{Im} \left(\frac{\Gamma_{12}}{M_{12}} \right) \right]. \quad (1.10)$$

To discuss CP violation in decay (see below), we need to consider decay amplitudes. The CP transformation law for a final state f is

$$\text{CP}|f\rangle = \omega_f|\bar{f}\rangle, \quad \text{CP}|\bar{f}\rangle = \omega_f^*|f\rangle, \quad (|\omega_f|) = 1. \quad (1.11)$$

For a final CP eigenstate $f = \bar{f} = f_{\text{CP}}$, the phase factor ω_f is replaced by $\eta_{f_{\text{CP}}} = \pm 1$, the CP eigenvalue of the final state. We define the decay amplitudes A_f and \bar{A}_f according to

$$A_f = \langle f|\mathcal{H}_d|B^0\rangle, \quad \bar{A}_f = \langle f|\mathcal{H}_d|\bar{B}^0\rangle, \quad (1.12)$$

where \mathcal{H}_d is the decay Hamiltonian.

To discuss CP violation in the interference of decays with and without mixing (see below), we introduce a complex quantity λ_f defined by

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (1.13)$$

We further define the CP transformation law for the quark fields in the Hamiltonian (a careful treatment of CP conventions can be found in [21]):

$$q \rightarrow \omega_q \bar{q}, \quad \bar{q} \rightarrow \omega_q^* q, \quad (|\omega_q| = 1). \quad (1.14)$$

The effective Hamiltonian that is relevant to M_{12} is of the form

$$H_{\text{eff}}^{\Delta b=2} \propto e^{+2i\phi_B} \left[\bar{d}\gamma^\mu(1 - \gamma_5)b \right]^2 + e^{-2i\phi_B} \left[\bar{b}\gamma^\mu(1 - \gamma_5)d \right]^2, \quad (1.15)$$

where $2\phi_B$ is a CP violating (weak) phase. (We use the Standard Model $V - A$ amplitude, but the results can be generalized to any Dirac structure.) For the B system, where $|\Gamma_{12}| \ll |M_{12}|$, this leads to

$$q/p = \omega_B \omega_b^* \omega_d e^{-2i\phi_B}. \quad (1.16)$$

(We implicitly assumed that the vacuum insertion approximation gives the correct sign for M_{12} . In general, there is a $\text{sign}(B_B)$ factor on the right hand side of (1.16) [22].) To

understand the phase structure of decay amplitudes, we take as an example the $b \rightarrow q\bar{q}d$ decay ($q = u$ or c). The decay Hamiltonian is of the form

$$H_d \propto e^{+i\phi_f} [\bar{q}\gamma^\mu(1 - \gamma_5)d] [\bar{b}\gamma_\mu(1 - \gamma_5)q] + e^{-i\phi_f} [\bar{q}\gamma^\mu(1 - \gamma_5)b] [\bar{d}\gamma_\mu(1 - \gamma_5)q], \quad (1.17)$$

where ϕ_f is the appropriate weak phase. (Again, for simplicity we use a $V - A$ structure, but the results hold for any Dirac structure.) Then

$$\bar{A}_{\bar{f}}/A_f = \omega_f \omega_B^* \omega_b \omega_d^* e^{-2i\phi_f}. \quad (1.18)$$

Eqs. (1.16) and (1.18) together imply that for a final CP eigenstate,

$$\lambda_{f\text{CP}} = \eta_{f\text{CP}} e^{-2i(\phi_B + \phi_f)}. \quad (1.19)$$

D. The Three Types of CP Violation in Meson Decays

There are three different types of CP violation in meson decays:

- (i) CP violation in mixing, which occurs when the two neutral mass eigenstate admixtures cannot be chosen to be CP-eigenstates;
- (ii) CP violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its CP-conjugate process have different magnitudes;
- (iii) CP violation in the interference of decays with and without mixing, which occurs in decays into final states that are common to B^0 and \bar{B}^0 . (It often occurs in combination with the other two types but there are cases when, to an excellent approximation, it is the only effect.)

(In cascade decays [23–26], there appears a fourth type of CP violation [27,28]. We do not discuss this type of CP violation here.)

(i) CP violation in mixing:

$$|q/p| \neq 1. \quad (1.20)$$

This results from the mass eigenstates being different from the CP eigenstates, and requires a relative phase between M_{12} and Γ_{12} . For the neutral B system, this effect could be observed through the asymmetries in semileptonic decays:

$$a_{\text{SL}} = \frac{\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow \ell^+ \nu X) - \Gamma(B_{\text{phys}}^0(t) \rightarrow \ell^- \nu X)}{\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow \ell^+ \nu X) + \Gamma(B_{\text{phys}}^0(t) \rightarrow \ell^- \nu X)}. \quad (1.21)$$

In terms of q and p ,

$$a_{\text{SL}} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \quad (1.22)$$

CP violation in mixing has been observed in the neutral K system ($\mathcal{R}e \, \varepsilon_K \neq 0$).

In the neutral B system, the effect is expected to be small, $\lesssim \mathcal{O}(10^{-2})$. The reason is that, model independently, the effect cannot be larger than $\mathcal{O}(\Delta\Gamma_B/\Delta m_B)$. The difference in width is produced by decay channels common to B^0 and \bar{B}^0 . The branching ratios for such channels are at or below the level of 10^{-3} . Since various channels contribute with differing signs, one expects that their sum does not exceed the individual level. Hence, we can safely assume that $\Delta\Gamma_B/\Gamma_B = \mathcal{O}(10^{-2})$. On the other hand, it is experimentally known that $\Delta m_B/\Gamma_B \approx 0.7$.

To calculate the deviation of $|q/p|$ from a pure phase (see (1.10)),

$$1 - \left| \frac{q}{p} \right| = \frac{1}{2} \mathcal{I}m \frac{\Gamma_{12}}{M_{12}}, \quad (1.23)$$

one needs to calculate M_{12} and Γ_{12} . This involves large hadronic uncertainties, in particular in the hadronization models for Γ_{12} .

(ii) CP violation in decay:

$$|\bar{A}_{\bar{f}}/A_f| \neq 1. \quad (1.24)$$

This appears as a result of interference among various terms in the decay amplitude, and will not occur unless at least two terms have different weak phases and different strong phases.

CP asymmetries in charged B decays,

$$a_f = \frac{\Gamma(B^+ \rightarrow f^+) - \Gamma(B^- \rightarrow f^-)}{\Gamma(B^+ \rightarrow f^+) + \Gamma(B^- \rightarrow f^-)}, \quad (1.25)$$

are purely an effect of CP violation in decay. In terms of the decay amplitudes,

$$a_{f^\pm} = \frac{1 - |\bar{A}_{f^-}/A_{f^+}|^2}{1 + |\bar{A}_{f^-}/A_{f^+}|^2}. \quad (1.26)$$

CP violation in decay has been observed in the neutral K system ($\mathcal{R}e \, \varepsilon'_K \neq 0$ [29–31]).

There are two types of phases that may appear in A_f and $\bar{A}_{\bar{f}}$. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\bar{A}_{\bar{f}}$ with opposite signs. In the Standard Model these phases occur only in the CKM matrix which is part of the electroweak sector of the theory, hence these are often called “weak phases”. The weak phase of any single term is convention dependent. However the difference between the weak phases in two different terms in A_f is convention independent because the phase rotations of the initial and final states are the same for every term. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Such phases do not violate CP, since they appear in A_f and $\bar{A}_{\bar{f}}$ with the same sign. Their origin is the possible contribution from intermediate on-shell states in the decay process, that is an absorptive part of an amplitude that has contributions from coupled channels. Usually the dominant rescattering is due to strong interactions and hence the designation “strong phases” for the phase shifts so induced. Again only the relative strong phases of different terms in a scattering amplitude have physical content, an overall phase rotation of the entire amplitude has no physical consequences.

Thus it is useful to write each contribution to A in three parts: its magnitude A_i ; its weak phase term $e^{i\phi_i}$; and its strong phase term $e^{i\delta_i}$. Then, if several amplitudes contribute to $B \rightarrow f$, we have

$$\left| \frac{\bar{A}_{\bar{f}}}{A_f} \right| = \left| \frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}} \right|. \quad (1.27)$$

The magnitude and strong phase of any amplitude involve long distance strong interaction physics, and our ability to calculate these from first principles is limited. Thus quantities that depend only on the weak phases are much cleaner than those that require knowledge

of the relative magnitudes or strong phases of various amplitude contributions, such as CP violation in decay. There is however a large literature and considerable theoretical effort that goes into the calculation of amplitudes and strong phases. In many cases we can only relate experiment to Standard Model parameters through such calculations. The techniques that are used are expected to be more accurate for B decays than for K decays, because of the larger B mass, but theoretical uncertainty remains significant. The calculations generally contain two parts. First the operator product expansion and QCD perturbation theory are used to write any underlying quark process as a sum of local quark operators with well-determined coefficients. Then the matrix elements of the operators between the initial and final hadron states must be calculated. This is where theory is weakest and the results are most model dependent. Ideally lattice calculations should be able to provide accurate determinations for the matrix elements, and in certain cases this is already true, but much remains to be done.

(iii) CP violation in the interference between decays with and without mixing:

$$|\lambda_{f_{\text{CP}}}| = 1, \quad \text{Im } \lambda_{f_{\text{CP}}} \neq 0. \quad (1.28)$$

Any $\lambda_{f_{\text{CP}}} \neq \pm 1$ is a manifestation of CP violation. The special case (1.28) isolates the effects of interest since both CP violation in decay (1.24) and in mixing (1.20) lead to $|\lambda_{f_{\text{CP}}}| \neq 1$. For the neutral B system, this effect can be observed by comparing decays into final CP eigenstates of a time-evolving neutral B state that begins at time zero as B^0 to those of the state that begins as \bar{B}^0 :

$$a_{f_{\text{CP}}} = \frac{\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{\text{CP}}) - \Gamma(B_{\text{phys}}^0(t) \rightarrow f_{\text{CP}})}{\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{\text{CP}}) + \Gamma(B_{\text{phys}}^0(t) \rightarrow f_{\text{CP}})}. \quad (1.29)$$

This time dependent asymmetry is given (for $|\lambda_{f_{\text{CP}}}| = 1$) by

$$a_{f_{\text{CP}}} = -\text{Im } \lambda_{f_{\text{CP}}} \sin(\Delta m_B t). \quad (1.30)$$

CP violation in the interference of decays with and without mixing has been observed for the neutral K system ($\text{Im } \varepsilon_K \neq 0$). It is expected to be an effect of $\mathcal{O}(1)$ in various B

decays. For such cases, the contribution from CP violation in mixing is clearly negligible. For decays that are dominated by a single CP violating phase (for example, $B \rightarrow \psi K_S$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$), so that the contribution from CP violation in decay is also negligible, $a_{f_{\text{CP}}}$ is cleanly interpreted in terms of purely electroweak parameters. Explicitly, $\mathcal{I}m\lambda_{f_{\text{CP}}}$ gives the difference between the phase of the $B - \bar{B}$ mixing amplitude ($2\phi_B$) and twice the phase of the relevant decay amplitude ($2\phi_f$) (see eq. (1.19)):

$$\mathcal{I}m\lambda_{f_{\text{CP}}} = -\eta_{f_{\text{CP}}} \sin[2(\phi_B + \phi_f)]. \quad (1.31)$$

E. Indirect vs. Direct CP Violation

The terms indirect CP violation and direct CP violation are commonly used in the literature. While various authors use these terms with different meanings, the most useful definition is the following:

- (i) **Indirect CP violation** refers to CP violation in meson decays where the CP violating phases can all be chosen to appear in $\Delta F = 2$ (mixing) amplitudes.
- (ii) **Direct CP violation** refers to CP violation in meson decays where some CP violating phases necessarily appear in $\Delta F = 1$ (decay) amplitudes.

Examining eqs. (1.20) and (1.7), we learn that CP violation in mixing is a manifestation of indirect CP violation. Examining eqs. (1.24) and (1.12), we learn that CP violation in decay is a manifestation of direct CP violation. Examining eqs. (1.28) and (1.13), we learn that the situation concerning CP violation in the interference of decays with and without mixing is more subtle. For any single measurement of $\mathcal{I}m\lambda_f \neq 0$, the relevant CP violating phase can be chosen by convention to reside in the $\Delta F = 2$ amplitude ($\phi_f = 0$, $\phi_B \neq 0$ in the notation of eq. (1.19)), and then we would call it indirect CP violation. Consider, however, the CP asymmetries for two different final CP eigenstates (for the same decaying meson), f_a and f_b . Then, a non-zero difference between $\mathcal{I}m\lambda_{f_a}$ and $\mathcal{I}m\lambda_{f_b}$ requires that there exists CP violation in $\Delta F = 1$ processes ($\phi_{f_a} - \phi_{f_b} \neq 0$), namely direct CP violation.

Experimentally, both direct and indirect CP violation have been established. Below we will see that ε_K signifies indirect CP violation while ε'_K signifies direct CP violation.

Theoretically, most models of CP violation (including the Standard Model) have predicted that both types of CP violation exist. There is, however, one class of models, that is *superweak models* [32–35], that predict only indirect CP violation. The measurement of $\varepsilon'_K \neq 0$ has excluded this class of models.

F. The ε_K and ε'_K Parameters

Historically, a different language from the one used by us has been employed to describe CP violation in $K \rightarrow \pi\pi$ and $K \rightarrow \pi\ell\nu$ decays. In this section we ‘translate’ the language of ε_K and ε'_K to our notations. Doing so will make it easy to understand which type of CP violation is related to each quantity.

The two CP violating quantities measured in neutral K decays are

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | \mathcal{H} | K_L \rangle}{\langle \pi^0 \pi^0 | \mathcal{H} | K_S \rangle}, \quad \eta_{+-} = \frac{\langle \pi^+ \pi^- | \mathcal{H} | K_L \rangle}{\langle \pi^+ \pi^- | \mathcal{H} | K_S \rangle}. \quad (1.32)$$

Define for $(ij) = (00)$ or $(+-)$

$$A_{ij} = \langle \pi^i \pi^j | \mathcal{H} | K^0 \rangle, \quad \bar{A}_{ij} = \langle \pi^i \pi^j | \mathcal{H} | \bar{K}^0 \rangle, \quad (1.33)$$

$$\lambda_{ij} = \left(\frac{q}{p} \right)_K \frac{\bar{A}_{ij}}{A_{ij}}. \quad (1.34)$$

Then

$$\eta_{00} = \frac{1 - \lambda_{00}}{1 + \lambda_{00}}, \quad \eta_{+-} = \frac{1 - \lambda_{+-}}{1 + \lambda_{+-}}. \quad (1.35)$$

The η_{00} and η_{+-} parameters get contributions from CP violation in mixing ($|(q/p)|_K \neq 1$) and from the interference of decays with and without mixing ($\text{Im}\lambda_{ij} \neq 0$) at $\mathcal{O}(10^{-3})$ and from CP violation in decay ($|\bar{A}_{ij}/A_{ij}| \neq 1$) at $\mathcal{O}(10^{-6})$.

There are two isospin channels in $K \rightarrow \pi\pi$ leading to final $(2\pi)_{I=0}$ and $(2\pi)_{I=2}$ states:

$$\langle \pi^0 \pi^0 | = \sqrt{\frac{1}{3}} \langle (\pi\pi)_{I=0} | - \sqrt{\frac{2}{3}} \langle (\pi\pi)_{I=2} |, \quad (1.36)$$

$$\langle \pi^+ \pi^- | = \sqrt{\frac{2}{3}} \langle (\pi\pi)_{I=0} | + \sqrt{\frac{1}{3}} \langle (\pi\pi)_{I=2} |. \quad (1.37)$$

The fact that there are two strong phases allows for CP violation in decay. The possible effects are, however, small (on top of the smallness of the relevant CP violating phases) because the final $I = 0$ state is dominant (this is the $\Delta I = 1/2$ rule). Defining

$$A_I = \langle (\pi\pi)_I | \mathcal{H} | K^0 \rangle, \quad \bar{A}_I = \langle (\pi\pi)_I | \mathcal{H} | \bar{K}^0 \rangle, \quad (1.38)$$

we have, experimentally,

$$|A_2/A_0| \approx 1/20. \quad (1.39)$$

Instead of η_{00} and η_{+-} we may define two combinations, ε_K and ε'_K , in such a way that the possible effects of CP violation in decay (mixing) are isolated into ε'_K (ε_K).

The experimental definition of the ε_K parameter is

$$\varepsilon_K \equiv \frac{1}{3}(\eta_{00} + 2\eta_{+-}). \quad (1.40)$$

To zeroth order in A_2/A_0 , we have $\eta_{00} = \eta_{+-} = \varepsilon_K$. However, the specific combination (1.40) is chosen in such a way that the following relation holds to *first* order in A_2/A_0 :

$$\varepsilon_K = \frac{1 - \lambda_0}{1 + \lambda_0}, \quad (1.41)$$

where

$$\lambda_0 = \left(\frac{q}{p} \right)_K \left(\frac{\bar{A}_0}{A_0} \right). \quad (1.42)$$

Since, by definition, only one strong channel contributes to λ_0 , there is indeed no CP violation in decay in (1.41). It is simple to show that $\mathcal{R}e \varepsilon_K \neq 0$ is a manifestation of CP violation in mixing while $\mathcal{I}m \varepsilon_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Since experimentally $\arg \varepsilon_K \approx \pi/4$, the two contributions

are comparable. It is also clear that $\varepsilon_K \neq 0$ is a manifestation of indirect CP violation: it could be described entirely in terms of a CP violating phase in the M_{12} amplitude.

The experimental definition of the ε'_K parameter is

$$\varepsilon'_K \equiv \frac{1}{3}(\eta_{+-} - \eta_{00}). \quad (1.43)$$

The theoretical expression is

$$\varepsilon'_K \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-}). \quad (1.44)$$

Obviously, any type of CP violation which is independent of the final state does not contribute to ε'_K . Consequently, there is no contribution from CP violation in mixing to (1.44). It is simple to show that $\mathcal{R}e \ \varepsilon'_K \neq 0$ is a manifestation of CP violation in decay while $\mathcal{I}m \ \varepsilon'_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Following our explanations in the previous section, we learn that $\varepsilon'_K \neq 0$ is a manifestation of direct CP violation: it requires $\phi_2 - \phi_0 \neq 0$ (where ϕ_I is the CP violating phase in the A_I amplitude defined in (1.38)).

II. CP VIOLATION IN THE STANDARD MODEL

A. Introduction

Within the Standard Model, CP violation can only arise from the Yukawa interactions:

$$-\mathcal{L}_Y = Y_{ij}^d \overline{Q_{Li}^I} \phi d_{Rj}^I + Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + Y_{ij}^\ell \overline{L_{Li}^I} \phi \ell_{Rj}^I. \quad (2.1)$$

The various fermion representations of $SU(3)_C \times SU(2)_L \times U(1)_Y$ are denoted here by

$$Q_i^I(3, 2)_{+1/6}, \quad \bar{u}_i^I(\bar{3}, 1)_{-2/3}, \quad \bar{d}_i^I(\bar{3}, 1)_{+1/3}, \quad L_i^I(1, 2)_{-1/2}, \quad \bar{\ell}_i^I(1, 1)_{+1}, \quad (2.2)$$

and the Higgs representation is $\phi(1, 2)_{+1/2}$ ($\tilde{\phi} = i\sigma_2 \phi^*$). There are 27 complex parameters in the three Yukawa matrices, but not all of them are physical. If the Yukawa couplings are switched off, the Standard Model has (in addition to a discrete CP symmetry) a global

$U(3)^5$ symmetry. But the subgroup of $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ remains a symmetry of the Standard Model even in the presence of non-zero Yukawa couplings. Since a 3×3 unitary matrix has three real and six imaginary parameters, we conclude that 15 real and 26 imaginary parameters in the Yukawa matrices are not physical, leaving twelve real and one imaginary physical parameters.

It is easy to identify the physical parameters in the mass basis. Nine of the real parameters are the charged fermion masses. All other parameters are related to the CKM matrix V_{CKM} , that is the quark mixing matrix that parametrizes the charged gauge boson interactions [36,1]:

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu (V_{\text{CKM}})_{ij} d_{Lj} W_\mu^\pm + \text{h.c.} \quad (2.3)$$

The unitary V_{CKM} can be parametrized with three real mixing angles and a single phase. The single irremovable phase in the CKM matrix is the only source CP violation within the Standard Model.

In the Wolfenstein parametrization of V_{CKM} , the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and η representing the CP violating phase [37]:

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (2.4)$$

The fact that there is a single CP violating parameter in the SM can be seen also in another useful way. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \quad (2.5)$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \quad (2.6)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (2.7)$$

Each of the three relations (2.5)–(2.7) requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are “the unitarity triangles”, though the term “unitarity triangle” is usually reserved for the relation (2.7) only. It is a surprising feature of the CKM matrix that all unitarity triangles are equal in area. For any choice of $i, j, k, l = 1, 2, 3$, one can define a quantity J according to [38]

$$\mathcal{I}m[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J \sum_{m,n=1}^3 \epsilon_{ikm}\epsilon_{jln}. \quad (2.8)$$

Then, the area of each unitarity triangle equals $|J|/2$ while the sign of J gives the direction of the complex vectors around the triangles. CP is violated in the Standard Model only if $J \neq 0$. The quantity J can then be taken as the CP violating parameter of the SM. The area of the triangles is then related to the size of the Standard Model CP violation. The relation between Jarlskog’s measure of CP violation J and the Wolfenstein parameters (λ, A, η) is given by

$$J \simeq \lambda^6 A^2 \eta. \quad (2.9)$$

The rescaled unitarity triangle is derived from (2.7) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η) .

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \sqrt{\rho^2 + \eta^2} = \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|, \quad R_t \equiv \sqrt{(1-\rho)^2 + \eta^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|. \quad (2.10)$$

The three angles of the unitarity triangle are denoted by α, β and γ [39]:

$$\alpha \equiv \arg \left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right], \quad \beta \equiv \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right], \quad \gamma \equiv \arg \left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]. \quad (2.11)$$

They are physical quantities and, we will soon see, can be independently measured by CP asymmetries in B decays.

To make predictions for future measurements of CP violating observables, we need to find the allowed ranges for the CKM phases. There are three ways to determine the CKM parameters (see *e.g.* [40]):

- (i) **Direct measurements** are related to SM tree level processes. At present, we have direct measurements of $|V_{ud}|$, $|V_{us}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$ and $|V_{tb}|$.
- (ii) **CKM Unitarity** ($V_{\text{CKM}}^\dagger V_{\text{CKM}} = \mathbf{1}$) relates the various matrix elements. At present, these relations are useful to constrain $|V_{td}|$, $|V_{ts}|$, $|V_{tb}|$ and $|V_{cs}|$.
- (iii) **Indirect measurements** are related to SM loop processes. At present, we constrain in this way $|V_{tb}V_{td}|$ (from Δm_B and Δm_{B_s}) and δ_{KM} (from ε_K).

When all available data is taken into account, we find [41–45]:

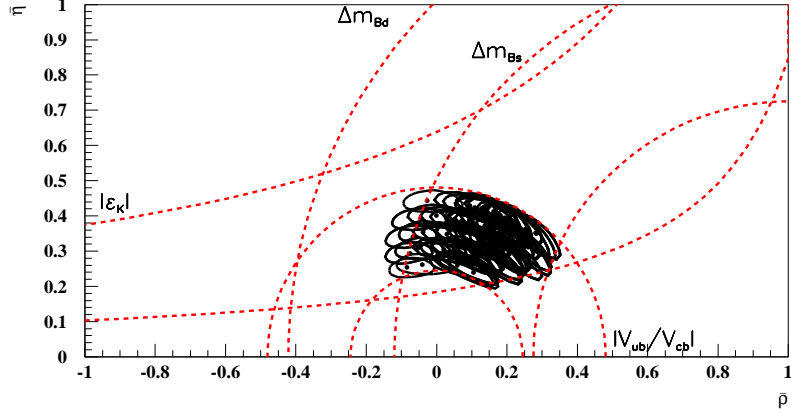
$$-0.15 \leq \rho \leq +0.35, \quad +0.20 \leq \eta \leq +0.45, \quad (2.12)$$

$$0.4 \leq \sin 2\beta \leq 0.8, \quad -0.9 \leq \sin 2\alpha \leq 1.0, \quad 0.23 \leq \sin^2 \gamma \leq 1.0. \quad (2.13)$$

Of course, there are correlations between the various parameters. The full information in the (ρ, η) and in the $(\sin 2\alpha, \sin 2\beta)$ plane is given in fig. 1.

Since the Standard Model contains only a single independent CP-violating phase, all possible CP-violating effects in this theory are very closely related. Consequently, the pattern of CP-violations in B decays is strongly constrained. The goal of B factories is to test whether this pattern occurs in Nature. If no new physics is discovered, we will have a much improved determination of the CKM parameters. The impact of measurements of CP asymmetries in B decays and of the rate of $K_L \rightarrow \pi\nu\nu$ is demonstrated in fig. 2.

(a)



(b)

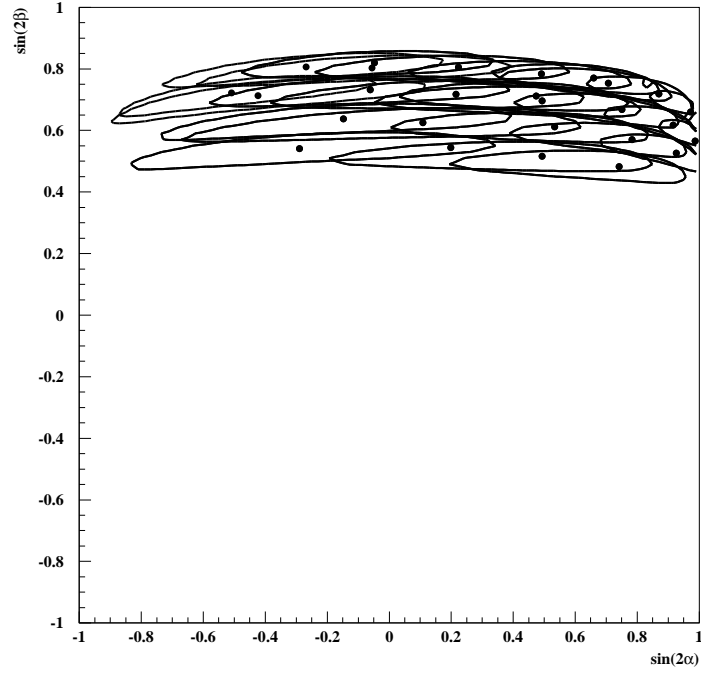


FIG. 1. The present allowed range (a) in the $\rho - \eta$ plane and (b) in the $\sin 2\alpha - \sin 2\beta$ plane using constraints from $|V_{cb}|$, $|V_{ub}/V_{cb}|$, Δm_{B_d} , ε_K and Δm_{B_s} . For the methods and the data used in this analysis, see ref. [45].

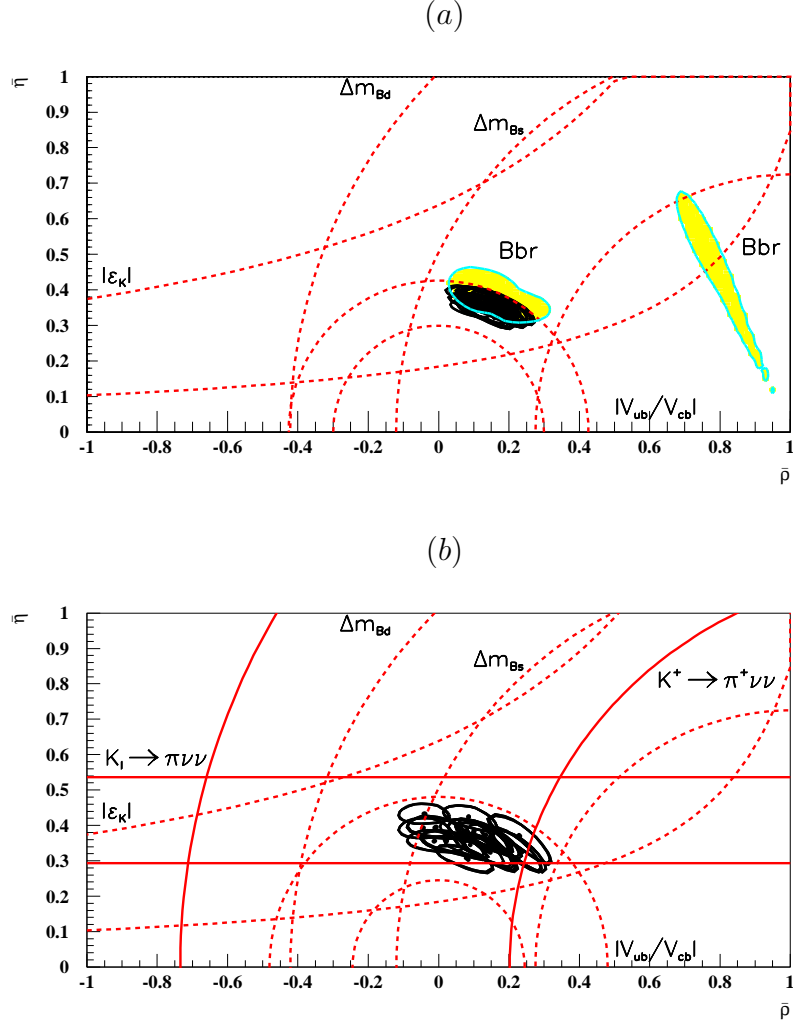


FIG. 2. (a) The effect of the CP asymmetries in B decays on the constraints in the $\rho - \eta$ plane. We use hypothetical ranges appropriate for 180 fb^{-1} integrated luminosity in the B -factories. (b) The effect of the $K \rightarrow \pi \nu \bar{\nu}$ measurements on the constraints in the $\rho - \eta$ plane. We use the hypothetical ranges $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \times 10^{-10}$, $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \times 10^{-11}$. For the methods and the data used in this analysis, see ref. [45].

B. CP Violation in Semileptonic Decays of Neutral B Mesons

In the B_d system we expect model independently that $\Gamma_{12} \ll M_{12}$. Moreover, within the SM and assuming that the box diagram (with a cut) is appropriate to estimate Γ_{12} , we can actually calculate the two quantities from quark diagrams [46]. The calculation gives

$$\frac{\Gamma_{12}}{M_{12}} = -\frac{3\pi}{2} \frac{1}{f_2(m_t^2/m_W^2)} \frac{m_b^2}{m_t^2} \left(1 + \frac{8}{3} \frac{m_c^2}{m_b^2} \frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} \right). \quad (2.14)$$

This confirms our order of magnitude estimate, $|\Gamma_{12}/M_{12}| \lesssim 10^{-2}$. The deviation of $|q/p|$ from unity is proportional to $\mathcal{I}m(\Gamma_{12}/M_{12})$ which is even further suppressed by another order of magnitude:

$$1 - \left| \frac{q}{p} \right| = \frac{1}{2} \mathcal{I}m \frac{\Gamma_{12}}{M_{12}} = \frac{4\pi}{f_2(m_t^2/m_W^2)} \frac{m_c^2}{m_t^2} \frac{J}{|V_{tb}V_{td}^*|^2} \sim 10^{-3}. \quad (2.15)$$

Note that the suppression comes from the (m_c^2/m_t^2) factor. The last term is the ratio of the area of the unitarity triangle to the length of one of its sides squared, so it is $\mathcal{O}(1)$. In contrast, for the B_s system, where (2.15) holds except that V_{td} is replaced by V_{ts} , there is an additional suppression from $J/|V_{tb}V_{ts}^*|^2 \sim 10^{-2}$ (see the corresponding unitarity triangle).

The above estimate of CP violation in mixing suffers from large uncertainties (of order 30% [19] or even higher [47]) related to the use of a quark diagram to describe Γ_{12} .

We remind the reader that (2.15) gives the leading contribution to the CP asymmetry in semileptonic decays:

$$a_{\text{SL}} \approx 2(1 - |q/p|). \quad (2.16)$$

The smallness of the predicted asymmetry will make its measurement a rather challenging task.

C. CP Violation in Hadronic Decays of Neutral B Mesons

In the previous subsection we estimated the effect of CP violation in mixing to be of $\mathcal{O}(10^{-3})$ within the Standard Model, and $\leq \mathcal{O}(|\Gamma_{12}/M_{12}|) \sim 10^{-2}$ model independently

(for a recent discussion, see [48]). In semileptonic decays, CP violation in mixing is the leading effect and therefore it can be measured through a_{SL} . In purely hadronic B decays, however, CP violation in decay and in the interference of decays with and without mixing is $\geq \mathcal{O}(10^{-2})$. We can therefore safely neglect CP violation in mixing in the following discussion and use

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \omega_B. \quad (2.17)$$

(From here on we omit the convention-dependent quark phases ω_q defined in eq. (1.14). Our final expressions for physical quantities are of course unaffected by such omission.)

A crucial question is then whether CP violation in decay is comparable to the CP violation in the interference of decays with and without mixing or negligible. In the first case, we can use the corresponding charged B decays to observe effects of CP violation in decay. In the latter case, CP asymmetries in neutral B decays are subject to clean theoretical interpretation: we will either have precise measurements of CKM parameters or be provided with unambiguous evidence for new physics. The question of the relative size of CP violation in decay can only be answered on a channel by channel basis, which is what we do in this section.

Most channels have contributions from both tree- and three types of penguin-diagrams, the latter classified according to the identity of the quark in the loop, as diagrams with different intermediate quarks may have both different strong phases and different weak phases [49]. On the other hand, the subdivision of tree processes into spectator, exchange and annihilation diagrams is unimportant in this respect since they all carry the same weak phase.

While quark diagrams can be easily classified in this way, the description of B decays is not so neatly divided into tree and penguin contributions once long distance physics effects are taken into account. Rescattering processes can change the quark content of the final state and confuse the identification of a contribution. There is no physical distinction between rescattered tree diagrams and long-distance contributions to the cuts of a penguin diagram.

While these issues complicate estimates of various rates, they can always be avoided in describing the weak phase structure of B -decay amplitudes. The decay amplitudes for $b \rightarrow q\bar{q}q'$ can always be written as a sum of three terms with definite CKM coefficients:

$$A(q\bar{q}q') = V_{tb}V_{tq'}^*P_{q'}^t + V_{cb}V_{cq'}^*(T_{c\bar{c}q'}\delta_{qc} + P_{q'}^c) + V_{ub}V_{uq'}^*(T_{u\bar{u}q'}\delta_{qu} + P_{q'}^u). \quad (2.18)$$

Here P and T denote contributions from tree and penguin diagrams, excluding the CKM factors. As they stand, the P terms are not well defined because of the divergences of the penguin diagrams. Only differences of penguin diagrams are finite and well defined. However already we see that diagrams that can be mixed by rescattering effects always appear with the same CKM coefficients and hence that a separation of these terms is not needed when discussing weak phase structure. Now it is useful to use eqs. (2.6) and (2.7) to eliminate one of the three terms, by writing its CKM coefficient as minus the sum of the other two.

In the case of $q\bar{q}s$ decays it is convenient to remove the $V_{tb}V_{ts}^*$ term. Then

$$A(c\bar{c}s) = V_{cb}V_{cs}^*(T_{c\bar{c}s} + P_s^c - P_s^t) + V_{ub}V_{us}^*(P_s^u - P_s^t), \quad (2.19)$$

$$A(u\bar{u}s) = V_{cb}V_{cs}^*(P_s^c - P_s^t) + V_{ub}V_{us}^*(T_{u\bar{u}s} + P_s^u - P_s^t), \quad (2.20)$$

$$A(s\bar{s}s) = V_{cb}V_{cs}^*(P_s^c - P_s^t) + V_{ub}V_{us}^*(P_s^u - P_s^t). \quad (2.21)$$

In these expressions only differences of penguin contributions occur, which makes the cancellation of the ultraviolet divergences of these diagrams explicit. Furthermore, the second term has a CKM coefficient that is much smaller, by $\mathcal{O}(\lambda^2)$, than the first. Hence this grouping is useful in classifying the expected CP violation in decay.

In the case of $q\bar{q}d$ decays the three CKM coefficients are of similar magnitude. The convention is then to retain the $V_{tb}V_{td}^*$ term because, in the Standard Model, the phase difference between this weak phase and half the mixing weak phase is zero. Thus only one unknown weak phase enters the calculation of the interference between decays with and without mixing. We can choose to eliminate which of the other terms does not have a tree

contribution. In the cases $q = s$ or d , since neither has a tree contribution either term can be removed. Thus we write

$$A(c\bar{c}d) = V_{tb}V_{td}^*(P_d^t - P_d^u) + V_{cb}V_{cd}^*(T_{c\bar{c}d} + P_d^c - P_d^u), \quad (2.22)$$

$$A(u\bar{u}d) = V_{tb}V_{td}^*(P_d^t - P_d^c) + V_{ub}V_{ud}^*(T_{u\bar{u}d} + P_d^u - P_d^c), \quad (2.23)$$

$$A(s\bar{s}d) = V_{tb}V_{td}^*(P_d^t - P_d^u) + V_{cb}V_{cd}^*(P_d^c - P_d^u). \quad (2.24)$$

Again only differences of penguin amplitudes occur. Furthermore the difference of penguin terms that occurs in the second term would vanish if the charm and up quark masses were equal, and thus is GIM suppressed [50]. However, even in modes with no tree contribution, $(s\bar{s}d)$, the interference of the two terms can still give significant CP violation.

The penguin processes all involve the emission of a neutral boson, either a gluon (strong penguins) or a photon or Z boson (electroweak penguins). Excluding the CKM coefficients, the ratio of the contribution from the difference between a top and light quark strong penguin diagram to the contribution from a tree diagram is of order

$$r_{PT} = \frac{P^t - P^{light}}{T_{q\bar{q}q'}} \approx \frac{\alpha_s}{12\pi} \ln \frac{m_t^2}{m_b^2}. \quad (2.25)$$

This is a factor of $\mathcal{O}(0.03)$. However this estimate does not include the effect of hadronic matrix elements, which are the probability factor to produce a particular final state particle content from a particular quark content. Since this probability differs for different kinematics, color flow and spin structures, it can be different for tree and penguin contributions and may partially compensate the coupling constant suppression of the penguin term. Recent CLEO results on $BR(B \rightarrow K\pi)$ and $BR(B \rightarrow \pi\pi)$ [51] suggest that the matrix element of penguin operators is indeed enhanced compared to that of tree operators. The enhancement could be by a factor of a few, leading to

$$r_{PT} \sim \lambda^2 - \lambda. \quad (2.26)$$

(Note that r_{PT} does not depend on the CKM parameters. We use powers of the Wolfenstein parameter λ to quantify our estimate for r_{PT} is order to simplify the comparison between the size of CP violation in decay and CP violation in the interference between decays with and without mixing.) Electroweak penguin difference terms are even more suppressed since they have an α_{EM} or α_W instead of the α_s factor in (2.25), but certain Z -contributions are enhanced by the large top quark mass and so can be non-negligible.

We thus classify B decays into four classes. Classes (i) and (ii) are expected to have relatively small CP violation in decay and hence are particularly interesting for extracting CKM parameters from interference of decays with and without mixing. In the remaining two classes, CP violation in decay could be significant and the neutral decay asymmetries cannot be cleanly interpreted in terms of CKM phases.

- (i) Decays dominated by a single term: $b \rightarrow c\bar{c}s$ and $b \rightarrow s\bar{s}s$. The Standard Model cleanly predicts very small CP violation in decay: $\mathcal{O}(\lambda^4 - \lambda^3)$ for $b \rightarrow c\bar{c}s$ and $\mathcal{O}(\lambda^2)$ for $b \rightarrow s\bar{s}s$. Any observation of large CP asymmetries in charged B decays for these channels would be a clue to physics beyond the Standard Model. The corresponding neutral modes have cleanly predicted relationships between CKM parameters and the measured asymmetry from interference between decays with and without mixing. The modes $B \rightarrow \psi K$ and $B \rightarrow \phi K$ are examples of this class.
- (ii) Decays with a small second term: $b \rightarrow c\bar{c}d$ and $b \rightarrow u\bar{u}d$. The expectation that penguin-only contributions are suppressed compared to tree contributions suggests that these modes will have small effects of CP violation in decay, of $\mathcal{O}(\lambda^2 - \lambda)$, and an approximate prediction for the relationship between measured asymmetries in neutral decays and CKM phases can be made. Examples here are $B \rightarrow DD$ and $B \rightarrow \pi\pi$.
- (iii) Decays with a suppressed tree contribution: $b \rightarrow u\bar{u}s$. The tree amplitude is suppressed by small mixing angles, $V_{ub}V_{us}$. The no-tree term may be comparable or even dominate and give large interference effects. An example is $B \rightarrow \rho K$.

- (iv) Decays with no tree contribution: $b \rightarrow s\bar{s}d$. Here the interference comes from penguin contributions with different charge 2/3 quarks in the loop and gives CP violation in decay that could be as large as 10% [52,53]. An example is $B \rightarrow KK$.

Note that if the penguin enhancement is significant, then some of the decay modes listed in class (ii) might actually fit better in class (iii). For example, it is possible that $b \rightarrow u\bar{u}d$ decays have comparable contributions from tree and penguin amplitudes. On the other hand, this would also mean that some modes listed in class (iii) could be dominated by a single penguin term. For such cases an approximate relationship between measured asymmetries in neutral decays and CKM phases can be made.

D. CP Violation in the Interference Between B Decays With and Without Mixing

Let us first discuss an example of class (i), $B \rightarrow \psi K_S$. A new ingredient in the analysis is the effect of $K - \bar{K}$ mixing. For decays with a single K_S in the final state, $K - \bar{K}$ mixing is essential because $B^0 \rightarrow K^0$ and $\bar{B}^0 \rightarrow \bar{K}^0$, and interference is possible only due to $K - \bar{K}$ mixing. This adds a factor of

$$\left(\frac{p}{q}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\omega_K^* \quad (2.27)$$

into (\bar{A}/A) . The quark subprocess in $\bar{B}^0 \rightarrow \psi \bar{K}^0$ is $b \rightarrow c\bar{c}s$ which is dominated by the W -mediated tree diagram:

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = \eta_{\psi K_S} \left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \right) \left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \right) \omega_B^*. \quad (2.28)$$

The CP-eigenvalue of the state is $\eta_{\psi K_S} = -1$. Combining (2.17) and (2.28), we find

$$\lambda(B \rightarrow \psi K_S) = - \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \right) \left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \right) \left(\frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*} \right) \implies \mathcal{I}m\lambda_{\psi K_S} = \sin(2\beta). \quad (2.29)$$

The second term in (2.19) is of order $\lambda^2 r_{PT}$ for this decay and thus eq. (2.29) is clean of hadronic uncertainties to $\mathcal{O}(10^{-3})$. Consequently, this measurement can give the theoretically cleanest determination of a CKM parameter, even cleaner than the determination

of $|V_{us}|$ from $K \rightarrow \pi \ell \nu$. (If $\text{BR}(K_L \rightarrow \pi \nu \bar{\nu})$ is measured, it will give a comparably clean determination of η .)

A second example of a theoretically clean mode in class (i) is $B \rightarrow \phi K_S$. The quark subprocess involves FCNC and cannot proceed via a tree level SM diagram. The leading contribution comes from penguin diagrams. The two terms in eq. (2.21) are now both differences of penguins but the second term is CKM suppressed and thus of $\mathcal{O}(\lambda^2)$ compared to the first. Thus CP violation in the decay is at most a few percent, and can be neglected in the analysis of asymmetries in this channel. The analysis is similar to the ψK_S case, and the asymmetry is proportional to $\sin(2\beta)$.

The same quark subprocesses give theoretically clean CP asymmetries also in B_s decays. These asymmetries are, however, very small since the relative phase between the mixing amplitude and the decay amplitudes (β_s defined below) is very small.

The best known example of class (ii) is $B \rightarrow \pi\pi$. The quark subprocess is $b \rightarrow u\bar{u}d$ which is dominated by the W -mediated tree diagram. Neglecting for the moment the second, pure penguin, term in eq. (2.23) we find

$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \eta_{\pi\pi} \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}} \omega_B^*. \quad (2.30)$$

The CP eigenvalue for two pions is $+1$. Combining (2.17) and (2.30), we get

$$\lambda(B \rightarrow \pi^+\pi^-) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \right) \left(\frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*} \right) \implies \mathcal{I}m\lambda_{\pi\pi} = \sin(2\alpha). \quad (2.31)$$

The pure penguin term in eq. (2.23) has a weak phase, $\arg(V_{td}^*V_{tb})$, different from the term with the tree contribution, so it modifies both $\mathcal{I}m\lambda_{\pi\pi}$ and (if there are non-trivial strong phases) $|\lambda_{\pi\pi}|$. The recent CLEO results mentioned above suggest that the penguin contribution to $B \rightarrow \pi\pi$ channel is significant, probably 10% or more. This then introduces CP violation in decay, unless the strong phases cancel (or are zero, as suggested by factorization arguments). The resulting hadronic uncertainty can be eliminated using isospin analysis [54]. This requires a measurement of the rates for the isospin-related channels $B^+ \rightarrow \pi^+\pi^0$ and $B^0 \rightarrow \pi^0\pi^0$ as well as the corresponding CP-conjugate processes. The rate for $\pi^0\pi^0$ is

expected to be small and the measurement is difficult, but even an upper bound on this rate can be used to limit the magnitude of hadronic uncertainties [55].

Related but slightly more complicated channels with the same underlying quark structure are $B \rightarrow \rho^0 \pi^0$ and $B \rightarrow a_1^0 \pi^0$. Again an analysis involving the isospin-related channels can be used to help eliminate hadronic uncertainties from CP violations in the decays [56,57].

Channels such as $\rho\rho$ and $a_1\rho$ could in principle also be studied, using angular analysis to determine the mixture of CP-even and CP-odd contributions.

The analysis of $B \rightarrow D^+ D^-$ proceeds along very similar lines. The quark subprocess here is $b \rightarrow c \bar{c} d$, and so the tree contribution gives

$$\lambda(B \rightarrow D^+ D^-) = \eta_{D^+ D^-} \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cd}^* V_{cb}}{V_{cd} V_{cb}^*} \right) \implies \mathcal{I}m \lambda_{DD} = -\sin(2\beta), \quad (2.32)$$

where we used $\eta_{D^+ D^-} = +1$. Again, there are hadronic uncertainties due to the pure penguin term in (2.22), but they are estimated to be small.

In all cases the above discussions have neglected the distinction between strong penguins and electroweak penguins. The CKM phase structure of both types of penguins is the same. The only place where this distinction becomes important is when an isospin argument is used to remove hadronic uncertainties due to penguin contributions. These arguments are based on the fact that gluons have isospin zero, and hence strong penguin processes have definite ΔI . Photons and Z -bosons on the other hand contribute to more than one ΔI transition and hence cannot be separated from tree terms by isospin analysis. In most cases electroweak penguins are small, typically no more than ten percent of the corresponding strong penguins and so their effects can safely be neglected. However in cases (iii) and (iv), where tree contributions are small or absent, their effects may need to be considered. (A full review of the role of electroweak penguins in B decays has been given in ref. [58].)

E. Unitarity Triangles

One can obtain an intuitive understanding of the Standard Model CP violation in the interference between decays with and without mixing by examining the unitarity triangles.

It is instructive to draw the three triangles, (2.5), (2.6) and (2.7), knowing the experimental values (within errors) for the various $|V_{ij}|$. In the first triangle (2.5), one side is of $\mathcal{O}(\lambda^5)$ and therefore much shorter than the other, $\mathcal{O}(\lambda)$, sides. In the second triangle (2.6), one side is of $\mathcal{O}(\lambda^4)$ and therefore shorter than the other, $\mathcal{O}(\lambda^2)$, sides. In the third triangle (2.7), all sides have lengths of $\mathcal{O}(\lambda^3)$. The first two triangles then almost collapse to a line while the third one is open.

Let us examine the CP asymmetries in the leading decays into final CP eigenstates. For the B mesons, the size of these asymmetries (*e.g.* $\mathcal{I}m\lambda_{\psi K_S}$) depends on β because it gives the difference between half the phase of the $B - \bar{B}$ mixing amplitude and the phase of the decay amplitudes. The form of the third unitarity triangle, (2.7), implies that $\beta = \mathcal{O}(1)$, which explains why these asymmetries are expected to be large.

It is useful to define the analog phases for the B_s meson, β_s , and the K meson, β_K :

$$\beta_s \equiv \arg \left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right], \quad \beta_K \equiv \arg \left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*} \right]. \quad (2.33)$$

The angles β_s and β_K can be seen to be the small angles of the second and first unitarity triangles, (2.6) and (2.5), respectively. This gives an intuitive understanding of why CP violation is small in the leading K decays (that is ε_K measured in $K \rightarrow \pi\pi$ decays) and is expected to be small in the leading B_s decays (*e.g.* $B_s \rightarrow \psi\phi$). Decays related to the short sides of these triangles are rare but could exhibit significant CP violation. Actually, the large angles in the (2.5) triangle are approximately β and $\pi - \beta$, which explains why CP violation in $K \rightarrow \pi\nu\bar{\nu}$ is related to β and expected to be large. The large angles in the (2.6) triangle are approximately γ and $\pi - \gamma$. This explains why the CP asymmetry in $B_s \rightarrow \rho K_S$ is related to γ and expected to be large. (Note, however, that this mode gets comparable contributions from penguin and tree diagrams and does not give a clean CKM measurement [53].)

III. CP VIOLATION BEYOND THE STANDARD MODEL

The Standard Model picture of CP violation is rather unique and highly predictive. In particular, we would like to point out the following features:

- (i) CP is broken explicitly.
- (ii) All CP violation arises from a single phase, that is δ_{KM} .
- (iii) The measured value of ε_K requires that δ_{KM} is of order one. (In other words, CP is not an approximate symmetry of the Standard Model.)
- (iv) The values of all other CP violating observables can be predicted. In particular, CP violation in $B \rightarrow \psi K_S$ (and similarly various other CP asymmetries in B decays), and in $K \rightarrow \pi \nu \bar{\nu}$ are expected to be of order one.

The commonly repeated statement that CP violation is one of the least tested aspects of the Standard Model is well demonstrated by the fact that none of the above features necessarily holds in the presence of New Physics. In particular, there are viable models of new physics (*e.g.* certain supersymmetric models) with the following features:

- (i) CP is broken spontaneously.
- (ii) There are many CP violating phases (even in the low energy effective theory).
- (iii) CP is an approximate symmetry, with all CP violating phases small (usually $10^{-3} \lesssim \phi_{\text{CP}} \lesssim 10^{-2}$).
- (iv) Values of CP violating observables can be predicted and could be very different from the Standard Model predictions (except, of course, ε_K). In particular, $\mathcal{I}m\lambda_{\psi K_S}$ and $a_{\pi \nu \bar{\nu}}$ could both be $\ll 1$.

We emphasize that various extensions of the SM modify its predictions of CP violation in various ways. The models of approximate CP described above are only one example of how the predictions can be dramatically violated.

To understand how the Standard Model predictions could be modified by New Physics, we will focus on CP violation in the interference between decays with and without mixing. As explained above, it is this type of CP violation which, due to its theoretical cleanliness, may give unambiguous evidence for New Physics most easily.

A. CP Violation as a Probe of Flavor Beyond the Standard Model

Let us consider five specific CP violating observables.

- (i) $\mathcal{I}m\lambda_{\psi K_S}$, the CP asymmetry in $B \rightarrow \psi K_S$. This measurement will cleanly determine the relative phase between the $B - \bar{B}$ mixing amplitude and the $b \rightarrow c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the Standard Model). The $b \rightarrow c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification by a phase θ_d :

$$\mathcal{I}m\lambda_{\psi K_S} = \sin[2(\beta + \theta_d)]. \quad (3.1)$$

- (ii) $\mathcal{I}m\lambda_{\phi K_S}$, the CP asymmetry in $B \rightarrow \phi K_S$. This measurement will cleanly determine the relative phase between the $B - \bar{B}$ mixing amplitude and the $b \rightarrow s\bar{s}s$ decay amplitude. The $b \rightarrow s\bar{s}s$ decay has only Standard Model penguin contributions and therefore is sensitive to new physics. We parametrize the modification of the decay amplitude by a phase θ_A [59]:

$$\mathcal{I}m\lambda_{\phi K_S} = \sin[2(\beta + \theta_d + \theta_A)]. \quad (3.2)$$

- (iii) $a_{\pi\nu\bar{\nu}}$, the CP violating ratio of $K \rightarrow \pi\nu\bar{\nu}$ decays:

$$a_{\pi\nu\bar{\nu}} = \frac{\Gamma(K_L \rightarrow \pi^0\nu\bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+\nu\bar{\nu})}. \quad (3.3)$$

This measurement will cleanly determine the relative phase between the $K - \bar{K}$ mixing amplitude and the $s \rightarrow d\nu\bar{\nu}$ decay amplitude. The experimentally measured small

value of ε_K requires that the phase of the $K - \bar{K}$ mixing amplitude is not modified from the Standard Model prediction. (More precisely, it requires that the phase in the mixing amplitude is very close to the phase in the $s \rightarrow d\bar{u}u$ decay amplitude.) On the other hand, the decay, which in the Standard Model is a loop process with small mixing angles, can be easily modified by new physics.

- (iv) $\mathcal{I}m(\lambda_{K^-\pi^+})$, the CP violating quantity in $D \rightarrow K^-\pi^+$ decay. The time-dependent ratios between the doubly-Cabibbo-suppressed and the Cabibbo-allowed $D \rightarrow K\pi$ decay rates are given by

$$\frac{\Gamma[D^0(t) \rightarrow K^+\pi^-]}{\Gamma[D^0(t) \rightarrow K^-\pi^+]} \simeq |\lambda|^2 + \frac{(\Delta m_D)^2}{4}t^2 + \mathcal{I}m(\lambda_{K^+\pi^-}^{-1})t, \quad (3.4)$$

$$\frac{\Gamma[\bar{D}^0(t) \rightarrow K^-\pi^+]}{\Gamma[\bar{D}^0(t) \rightarrow K^+\pi^-]} \simeq |\lambda|^2 + \frac{(\Delta m_D)^2}{4}t^2 + \mathcal{I}m(\lambda_{K^-\pi^+})t, \quad (3.5)$$

where we used the fact that CP violation in mixing is expected to be small and, consequently,

$$|\lambda_{K^+\pi^-}^{-1}| = |\lambda_{K^-\pi^+}| \equiv \lambda. \quad (3.6)$$

The ratio

$$a_{D \rightarrow K\pi} = \frac{\mathcal{I}m(\lambda_{K^-\pi^+})}{|\lambda_{K^-\pi^+}|} \quad (3.7)$$

depends on the relative phase between the $D - \bar{D}$ mixing amplitude and the $c \rightarrow d\bar{s}u$ decay amplitude. Within the Standard Model, this decay channel is tree level. It is unlikely that it is affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics [60].

- (v) d_N , the electric dipole moment of the neutron. We did not discuss this quantity so far because, unlike CP violation in meson decays, flavor changing couplings are not necessary for d_N . In other words, the CP violation that induces d_N is *flavor diagonal*. It does in general get contributions from flavor changing physics, but it could be

induced by sectors that are flavor blind. Within the Standard Model (and ignoring the strong CP angle θ_{QCD}), the contribution from δ_{KM} arises at the three loop level and is at least six orders of magnitude below the experimental bound [61] d_N^{exp} ,

$$d_N^{\text{exp}} = 1.1 \times 10^{-25} \text{ e cm.} \quad (3.8)$$

The various CP violating observables discussed above are sensitive then to new physics in the mixing amplitudes for the $B - \bar{B}$ and $D - \bar{D}$ systems, in the decay amplitudes for $b \rightarrow s\bar{s}s$ and $s \rightarrow d\nu\bar{\nu}$ channels and to flavor diagonal CP violation. If information about all these processes becomes available and deviations from the Standard Model predictions are found, we can ask rather detailed questions about the nature of the new physics that is responsible to these deviations:

- (i) Is the new physics related to the down sector? the up sector? both?
- (ii) Is the new physics related to $\Delta B = 1$ processes? $\Delta B = 2$? both?
- (iii) Is the new physics related to the third generation? to all generations?
- (iv) Are the new sources of CP violation flavor changing? flavor diagonal? both?

It is no wonder then that with such rich information, flavor and CP violation provide an excellent probe of new physics.

IV. SUPERSYMMETRY

A generic supersymmetric extension of the Standard Model contains a host of new flavor and CP violating parameters. (For reviews on supersymmetry see refs. [62–69]. The following chapter is based on [70].) The requirement of consistency with experimental data provides strong constraints on many of these parameters. For this reason, the physics of flavor and CP violation has had a profound impact on supersymmetric model building. A discussion of CP violation in this context can hardly avoid addressing the flavor problem itself. Indeed,

many of the supersymmetric models that we analyze below were originally aimed at solving flavor problems.

As concerns CP violation, one can distinguish two classes of experimental constraints. First, bounds on nuclear and atomic electric dipole moments determine what is usually called the *supersymmetric CP problem*. Second, the physics of neutral mesons and, most importantly, the small experimental value of ε_K pose the *supersymmetric ε_K problem*. In the next two subsections we describe the two problems. Then we describe various supersymmetric flavor problems and the ways in which they address the supersymmetric CP problem.

Before turning to a detailed discussion, we define two scales that play an important role in supersymmetry: Λ_S , where the soft supersymmetry breaking terms are generated, and Λ_F , where flavor dynamics takes place. When $\Lambda_F \gg \Lambda_S$, it is possible that there are no genuinely new sources of flavor and CP violation. This leads to models with exact universality, which we discuss in section IV.C. When $\Lambda_F \lesssim \Lambda_S$, we do not expect, in general, that flavor and CP violation are limited to the Yukawa matrices. One way to suppress CP violation would be to assume that CP is an approximate symmetry of the full theory (namely, CP violating phases are all small). We discuss this scenario in section IV.D. Another option is to assume that, similarly to the Standard Model, CP violating phases are large, but their effects are screened, possibly by the same physics that explains the various flavor puzzles. Such models, with Abelian or non-Abelian horizontal symmetries, are described in section IV.E. It is also possible that CP violating effects are suppressed because squarks are heavy. This scenario is also discussed in section IV.E. Some concluding comments are given in section IV.F.

A. The Supersymmetric CP Problem

One aspect of supersymmetric CP violation involves effects that are flavor preserving. Then, for simplicity, we describe this aspect in a supersymmetric model without additional flavor mixings, *i.e.* the minimal supersymmetric standard model (MSSM) with universal sfermion masses and with the trilinear SUSY-breaking scalar couplings proportional to the

corresponding Yukawa couplings. (The generalization to the case of non-universal soft terms is straightforward.) In such a constrained framework, there are four new phases beyond the two phases of the Standard Model (δ_{KM} and θ_{QCD}). One arises in the bilinear μ -term of the superpotential,

$$W = \mu H_u H_d, \quad (4.1)$$

while the other three arise in the soft supersymmetry breaking parameters $m_{\tilde{g}}$ (the gaugino mass), A (the trilinear scalar coupling) and m_{12}^2 (the bilinear scalar coupling):

$$\mathcal{L} = -\frac{1}{2}m_{\tilde{g}}\tilde{g}\tilde{g} - A(Y^u Q H_u \bar{u} - Y^d Q H_d \bar{d} - Y^e L H_d \bar{\ell}) - m_{12}^2 H_u H_d + \text{h.c.}, \quad (4.2)$$

where \tilde{g} are the gauginos and Y are Yukawa matrices. Only two combinations of the four phases are physical [71,72]. In the absence of (4.1) and (4.2), there are two additional global $U(1)$ symmetries in the MSSM, an R symmetry and a Peccei-Quinn symmetry. This means that one could treat the various dimensionful parameters in (4.1) and (4.2) as spurions which break the symmetries, thus deriving selection rules. The appropriate charge assignments are:

	$m_{\tilde{g}}$	A	m_{12}^2	μ	H_u	H_d	$Q\bar{u}$	$Q\bar{d}$	$L\bar{\ell}$
$U(1)_{\text{PQ}}$	0	0	-2	-2	1	1	-1	-1	-1
$U(1)_{\text{R}}$	-2	-2	-2	0	1	1	1	1	1

(4.3)

Physical observables can only depend on combinations of the dimensionful parameters that are neutral under both $U(1)$'s. There are three such independent combinations: $m_{\tilde{g}}\mu(m_{12}^2)^*$, $A\mu(m_{12}^2)^*$ and $A^*m_{\tilde{g}}$. However, only two of their phases are independent, say

$$\phi_A = \arg(A^*m_{\tilde{g}}), \quad \phi_B = \arg(m_{\tilde{g}}\mu(m_{12}^2)^*). \quad (4.4)$$

In the more general case of non-universal soft terms there is one independent phase ϕ_{A_i} for each quark and lepton flavor. Moreover, complex off-diagonal entries in the sfermion mass matrices may represent additional sources of CP violation.

The most significant effect of ϕ_A and ϕ_B is their contribution to electric dipole moments (EDMs). For example, the contribution from one-loop gluino diagrams to the down quark EDM is given by [73,74]:

$$d_d = M_d \frac{e\alpha_3}{18\pi\tilde{m}^4} (|Am_{\tilde{g}}| \sin \phi_A + \tan \beta |\mu m_{\tilde{g}}| \sin \phi_B), \quad (4.5)$$

where we have taken $m_Q^2 \sim m_D^2 \sim m_{\tilde{g}}^2 \sim \tilde{m}^2$, for left- and right-handed squark and gluino masses. We define, as usual, $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$. Similar one-loop diagrams give rise to chromoelectric dipole moments. The electric and chromoelectric dipole moments of the light quarks (u, d, s) are the main source of d_N (the EDM of the neutron), giving [75]

$$d_N \sim 2 \left(\frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \sin \phi_{A,B} \times 10^{-23} \text{ e cm} \quad (4.6)$$

where, as above, \tilde{m} represents the overall SUSY scale. The present experimental bound, $d_N < 1.1 \times 10^{-25} \text{ e cm}$ [76,77], is then violated for $\mathcal{O}(1)$ phases, unless the masses of superpartners are above $\mathcal{O}(1 \text{ TeV})$. Alternatively for light SUSY masses, the new phases should be $< \mathcal{O}(10^{-2})$. Notice however that one may consider the actual bound weaker than this, due to the theoretical uncertainty in the estimate of the hadronic matrix elements that lead to eq. (4.6) [78]. With this caveat, whether the phases are small or squarks are heavy, a fine-tuning of order 10^{-2} seems to be required, in general, to avoid too large a d_N . This is *the Supersymmetric CP Problem* [73,74,79,80].

In addition to d_N , the SUSY CP phases contribute to atomic and nuclear EDMs (see a detailed discussion in ref. [75]). The former are also sensitive to phases in the leptonic sector. The latter give additional constraints on the quark sector phases. For instance, the bound on the nuclear EDM of ^{199}Hg is comparable to the one given by d_N . In practice, these additional bounds on SUSY CP phases are not stronger than those from d_N (at least in the quark sector). However, since there are significant theoretical uncertainties in the calculation of nuclear EDMs, it is important to measure as many as possible of them to obtain more reliable bounds.

B. The Supersymmetric ε_K Problem

The contribution to the CP violating ε_K parameter in the neutral K system is dominated by diagrams involving Q and \bar{d} squarks in the same loop [81–85]. The corresponding

effective four-fermi operator involves fermions of both chiralities, so that its matrix elements are enhanced by $\mathcal{O}(m_K/m_s)^2$ compared to the chirality conserving operators. For $m_{\tilde{g}} \simeq m_Q \simeq m_D = \tilde{m}$ (our results depend only weakly on this assumption) and focusing on the contribution from the first two squark families, one gets (we use the results in ref. [85])

$$\varepsilon_K = \frac{5}{162\sqrt{2}} \frac{\alpha_3^2 f_K^2 m_K}{\tilde{m}^2 \Delta m_K} \left[\left(\frac{m_K}{m_s + m_d} \right)^2 + \frac{3}{25} \right] \text{Im} \left\{ \frac{(\delta m_Q^2)_{12}}{m_Q^2} \frac{(\delta m_D^2)_{12}}{m_D^2} \right\}, \quad (4.7)$$

where $(\delta m_{Q,D}^2)_{12}$ are the off diagonal entries in the squark mass matrices in a basis where the down quark mass matrix and the gluino couplings are diagonal. These flavor violating quantities are often written as $(\delta m_{Q,D}^2)_{12} = V_{11}^{Q,D} \delta m_{Q,D}^2 V_{21}^{Q,D*}$, where $\delta m_{Q,D}^2$ is the mass splitting among the squarks and $V^{Q,D}$ are the gluino coupling mixing matrices in the mass eigenbasis of quarks and squarks. Note that CP would be violated even if there were two families only [86]. (There are also contributions involving the third family squarks via the (13) and (23) mixings. In some cases the third family contribution actually dominates.)

Using the experimental value of ε_K , we get

$$\frac{(\Delta m_K \varepsilon_K)^{\text{SUSY}}}{(\Delta m_K \varepsilon_K)^{\text{EXP}}} \sim 10^7 \left(\frac{300 \text{ GeV}}{\tilde{m}} \right)^2 \left(\frac{m_{Q_2}^2 - m_{Q_1}^2}{m_Q^2} \right) \left(\frac{m_{D_2}^2 - m_{D_1}^2}{m_D^2} \right) |K_{12}^{dL} K_{12}^{dR}| \sin \phi, \quad (4.8)$$

where ϕ is the CP violating phase. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$, $\delta m_{Q,D}^2/m_{Q,D}^2 = \mathcal{O}(1)$, $K_{ij}^{Q,D} = \mathcal{O}(1)$ and $\sin \phi = \mathcal{O}(1)$. Then the constraint (4.8) is generically violated by about seven orders of magnitude. (Four-fermi operators with same chirality fermions give a smaller effect. The resulting ε_K -bounds are therefore weaker by about one order of magnitude.)

Eq. (4.8) also shows what are the possible ways to solve the supersymmetric ε_K problem:

- (i) Heavy squarks: $\tilde{m} \gg 300 \text{ GeV}$;
- (ii) Universality: $\delta m_{Q,D}^2 \ll m_{Q,D}^2$;
- (iii) Alignment: $|K_{12}^d| \ll 1$;
- (iv) Approximate CP: $\sin \phi \ll 1$.

The d_N problem (see eq. (4.6)) is solved by either heavy squarks or approximate CP.

C. Exact Universality

Both supersymmetric CP problems are solved if, at the scale Λ_S , the soft supersymmetry breaking terms are universal and the genuine SUSY CP phases $\phi_{A,B}$ vanish. Then the Yukawa matrices represent the only source of flavor and CP violation which is relevant in low energy physics. This situation can naturally arise when supersymmetry breaking is mediated by gauge interactions at a scale $\Lambda_S \ll \Lambda_F$ [87–90]. In the simplest scenarios, the A -terms and the gaugino masses are generated by the same SUSY and $U(1)_R$ breaking source (see eq. (4.3)). Thus, up to very small effects due to the *standard* Yukawa matrices, $\arg(A) = \arg(m_{\tilde{g}})$ so that ϕ_A vanishes. In specific models also ϕ_B vanishes in a similar way [88,90]. It is also possible that similar boundary conditions occur when supersymmetry breaking is communicated to the observable sector up at the Planck scale [91–95]. The situation in this case seems to be less under control from the theoretical point of view. Dilaton dominance in SUSY breaking, though, seems a very interesting direction to explore [96,97].

The most important implication of this type of boundary conditions for soft terms, which we refer to as *exact universality* [98,99], is the existence of the SUSY analogue of the GIM mechanism which operates in the SM. The CP violating phase of the CKM matrix can feed into the soft terms via Renormalization Group (RG) evolution only with a strong suppression from light quark masses [71].

With regard to the supersymmetric CP problem, gluino diagrams contribute to quark EDMs as in eq. (4.5), but with a highly suppressed effective phase, *e.g.*

$$\phi_{A_d} \sim (t_S/16\pi^2)^4 Y_t^4 Y_c^2 Y_b^2 J. \quad (4.9)$$

Here $t_S = \log(\Lambda_S/M_W)$ arises from the RG evolution from Λ_S to the electroweak scale, the Y_i 's are quark Yukawa couplings (in the mass basis), and $J \simeq 2 \times 10^{-5}$ is defined in eq. (2.8). A similar contribution comes from chargino diagrams. The resulting EDM is $d_N \lesssim 10^{-31} e \text{ cm}$. This maximum can be reached only for very large $\tan\beta \sim 60$ while,

for small $\tan\beta \sim 1$, d_N is about 5 orders of magnitude smaller. This range of values for d_N is much below the present ($\sim 10^{-25}$ e cm) and foreseen ($\sim 10^{-28}$ e cm) experimental sensitivities [100–103].

With regard to the supersymmetric ε_K problem, the contribution to ε_K is proportional to $\mathcal{I}m(V_{td}V_{ts}^*)^2 Y_t^4 (t_S/16\pi^2)^2$, giving the same GIM suppression as in the SM. This contribution turns out to be small [71]:

$$|\varepsilon_K^{\text{SUSY}}| \sim 6 \times 10^{-6} \left[\frac{J \mathcal{R}e(V_{td}V_{ts}^*)}{10^{-8}} \right] \left[\frac{300 \text{ GeV}}{\tilde{m}} \right]^2 \left[\frac{\ln(\Lambda_S/m_W)}{5} \right]^2. \quad (4.10)$$

The value $t_S = 5$ is typical to gauge mediated supersymmetry breaking, but (4.10) remains negligible for any scale $\Lambda_S \lesssim M_{\text{Pl}}$ (namely $t_S \lesssim 35$). The supersymmetric contribution to $D - \bar{D}$ mixing is similarly small and we expect no observable effects.

For the B_d and B_s systems, the largest SUSY contribution to the mixing comes from box diagrams with intermediate charged Higgs and the up quarks. It can be up to $\mathcal{O}(0.2)$ of the SM amplitude for $\Lambda_S = M_{\text{Pl}}$ and $\tan\beta = \mathcal{O}(1)$ [104–107], and much smaller for large $\tan\beta$. The contribution is smaller in models of gauge mediated SUSY breaking where the mass of the charged Higgs boson is typically $\gtrsim 300$ GeV [89] and $t_S \sim 5$. The SUSY contributions to $B_s - \bar{B}_s$ and $B_d - \bar{B}_d$ mixing are, to a good approximation, proportional to $(V_{tb}V_{ts}^*)^2$ and $(V_{tb}V_{td}^*)^2$, respectively, like in the SM. Then, regardless of the size of these contributions, the relation $\Delta m_{B_d}/\Delta m_{B_s} \sim |V_{td}/V_{ts}|^2$ and the CP asymmetries in neutral B decays into final CP eigenstates are the same as in the SM.

D. Approximate CP Symmetry

Both supersymmetric CP problems are solved if CP is an approximate symmetry, broken by a small parameter of order 10^{-3} . This is one of the possible solutions to CP problems in the class of supersymmetric models with $\Lambda_F \lesssim \Lambda_S$, where the soft masses are generically not universal, so that we do not expect flavor and CP violation to be limited to the Yukawa matrices. (Of course, some mechanism has also to suppress the real part of the $\Delta S = 2$

amplitude by a sufficient amount.) Most models where soft terms arise at the Planck scale ($\Lambda_S \sim M_{\text{Pl}}$) belong to this class.

If CP is an approximate symmetry, we expect also the SM phase δ_{KM} to be $\ll 1$. Then the standard box diagrams cannot account for ε_K which should arise from another source. In supersymmetry with non-universal soft terms, the source could be diagrams involving virtual superpartners, mainly squark-gluino box diagrams. Let us call $(M_{12}^K)^{\text{SUSY}}$ the supersymmetric contribution to the $K - \bar{K}$ mixing amplitude. Then the requirements $\text{Re}(M_{12}^K)^{\text{SUSY}} \lesssim \Delta m_K$ and $\text{Im}(M_{12}^K)^{\text{SUSY}} \sim \varepsilon_K \Delta m_K$ imply that the generic CP phases are $\geq \mathcal{O}(\varepsilon_K) \sim 10^{-3}$.

Of course, d_N constrains the relevant CP violating phases to be $\lesssim 10^{-2}$. If all phases are of the same order, then d_N must be just below or barely compatible with the present experimental bound. A signal should definitely be found if the accuracy is increased by two orders of magnitude.

The main phenomenological implication of these scenarios is that CP asymmetries in B meson decays are small, perhaps $\mathcal{O}(\varepsilon_K)$, rather than $\mathcal{O}(1)$ as expected in the SM. Also the ratio $a_{\pi\nu\bar{\nu}}$ (see (3.3)) is very small, in contrast to the Standard Model where it is expected to be of $\mathcal{O}(\sin^2 \beta)$. Explicit models of approximate CP were presented in refs. [108–110].

The fact that the Standard Model and the models of approximate CP are both viable at present is related to the fact that the mechanism of CP violation has not really been tested experimentally. The only measured CP violating observable, that is ε_K , is small. Its smallness could be related to the ‘accidental’ smallness of CP violation for the first two quark generations, as is the case in the Standard Model, or to CP being an approximate symmetry, as is the case in the models discussed here. Future measurements, particularly of processes where the third generation plays a dominant role (such as $a_{\psi K_S}$ or $a_{\pi\nu\bar{\nu}}$), will easily distinguish between the two scenarios. While the Standard Model predicts large CP violating effects for these processes, approximate CP would suppress them too.

The distinction between the Standard Model and Supersymmetry could also be made – though less easily – in measurements of CP violation in neutral D decays and of the

electric dipole moments of the neutron. Here, the GIM mechanism of the Standard Model is so efficient that CP violating effects are unobservable in both cases. In contrast, the flavor breaking in supersymmetry might be much stronger, and then the approximate CP somewhat suppresses the effects but to a level which is perhaps still observable.

E. Approximate Horizontal Symmetries

Another option is to assume that, similarly to the Standard Model, CP violating phases are large, but their effects are screened, possibly by the same physics that explains the various flavor puzzles. This usually requires Abelian or non-Abelian horizontal symmetries. Two ingredients play a major role here: selection rules that come from the symmetry and holomorphy of Yukawa and A -terms that comes from the supersymmetry. With Abelian symmetries, the screening mechanism is provided by *alignment* [111–113], whereby the mixing matrices for gaugino couplings have very small mixing angles, particularly for the first two down squark generations. With non-Abelian symmetries, the screening mechanism is *approximate universality*, where squarks of the two families fit into an irreducible doublet and are, therefore, approximately degenerate [114–122]. In all of these models, it is difficult to avoid $d_N \gtrsim 10^{-28}$ e cm.

As far as the third generation is concerned, the signatures of Abelian and non-Abelian models are similar. In particular, they allow observable deviations from the SM predictions for CP asymmetries in B decays. In some cases, non-Abelian models give relations between CKM parameters and consequently predict strong constraints on these CP asymmetries.

For the two light generations, only alignment allows interesting effects. In particular, it predicts large CP violating effects in $D - \bar{D}$ mixing [111,112]. Thus, it allows for $a_{D \rightarrow K\pi} = \mathcal{O}(1)$ and, in particular, for $\mathcal{I}m(\lambda_{K^+\pi^-}^{-1}) \neq \mathcal{I}m(\lambda_{K^-\pi^+})$ which will signify new CP violating phases (see eqs. (3.4)).

Finally, it is possible that CP violating effects are suppressed because squarks are heavy. If the masses of the first and second generations squarks m_i are larger than the other soft

masses, $m_i^2 \sim 100 \tilde{m}^2$ then the Supersymmetric CP problem is solved and the ε_K problem is relaxed (but not eliminated) [117,114]. This does not necessarily lead to naturalness problems, since these two generations are almost decoupled from the Higgs sector.

Notice though that, with the possible exception of $m_{\tilde{b}_R}^2$, third family squark masses cannot naturally be much above m_Z^2 . If the relevant phases are of $O(1)$, the main contribution to d_N comes from the third family via the two-loop induced three-gluon operator [124], and it is roughly at the present experimental bound when $m_{\tilde{t}_{L,R}} \sim 100 \text{ GeV}$.

Models with the first two squark generations heavy have their own signatures of CP violation in neutral meson mixing [125]. The mixing angles relevant to $D - \bar{D}$ mixing are similar, in general, to those of models of alignment (if alignment is invoked to explain Δm_K with $m_{\tilde{Q},D}^2 \lesssim 20 \text{ TeV}$). However, since the \tilde{u} and \tilde{c} squarks are heavy, the contribution to $D - \bar{D}$ mixing is one to two orders of magnitude below the experimental bound. This may lead to the interesting situation that $D - \bar{D}$ mixing will first be observed through its CP violating part [126]. In the neutral B system, $O(1)$ shifts from the Standard Model predictions of CP asymmetries in the decays to final CP eigenstates are possible. This can occur even when the squarks masses of the third family are $\sim 1 \text{ TeV}$ [127], since now mixing angles can naturally be larger than in the case of horizontal symmetries (alignment or approximate universality).

F. Some Concluding Comments

The conclusion from our discussion of supersymmetric flavor models is that measurements of CP violation will provide us with an excellent probe of the flavor and CP structure of supersymmetry. This is clearly demonstrated in Table I.

The unique features of CP violation are well demonstrated by examining the CP asymmetry in $B \rightarrow \psi K_S$, $\text{Im} \lambda_{\psi K_S}$, and CP violation in $K \rightarrow \pi \nu \bar{\nu}$, $\text{Im} \lambda_{\pi \nu \bar{\nu}}$. Model independently, $\text{Im} \lambda_{\psi K_S}$ measures the relative phase between the $B - \bar{B}$ mixing amplitude and the $b \rightarrow c \bar{c} d$ decay amplitude (more precisely, the $b \rightarrow c \bar{c} s$ decay amplitude times the $K - \bar{K}$ mixing am-

Model	d_N/d_N^{exp}	θ_d	θ_A	$a_{D^0 \rightarrow K^- \pi^+}$	$a_{K \rightarrow \pi \nu \bar{\nu}}$
Standard Model	$\lesssim 10^{-6}$	0	0	0	$\mathcal{O}(1)$
Exact Universality	$\lesssim 10^{-6}$	0	0	0	= SM
Approximate CP	$\sim 10^{-1}$	$-\beta$	0	$\mathcal{O}(10^{-3})$	$\mathcal{O}(10^{-5})$
Alignment	$\gtrsim 10^{-3}$	$\mathcal{O}(0.2)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\approx \text{SM}$
Approx. Universality	$\gtrsim 10^{-2}$	$\mathcal{O}(0.2)$	$\mathcal{O}(1)$	0	$\approx \text{SM}$
Heavy Squarks	$\sim 10^{-1}$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(10^{-2})$	$\approx \text{SM}$

TABLE I. CP violating observables in various classes of Supersymmetric flavor models.

plitude), while $\mathcal{I}m\lambda_{\pi\nu\bar{\nu}}$ measures the relative phase between the $K - \bar{K}$ mixing amplitude and the $s \rightarrow d\nu\bar{\nu}$ decay amplitude. We would like to emphasize the following three points:

- (i) *The two measurements are theoretically clean to better than $\mathcal{O}(10^{-2})$.* Thus they can provide the most accurate determination of CKM parameters.
- (ii) *As concerns CP violation, the Standard Model is a uniquely predictive model.* In particular, it predicts that the seemingly unrelated $\mathcal{I}m\lambda_{\psi K_S}$ and $\mathcal{I}m\lambda_{\pi\nu\bar{\nu}}$ measure the same parameter, that is the angle β of the unitarity triangle.
- (iii) *In the presence of New Physics, there is in general no reason for a relation between $\mathcal{I}m\lambda_{\psi K_S}$ and $\mathcal{I}m\lambda_{\pi\nu\bar{\nu}}$.* Therefore, a measurement of both will provide a sensitive probe of New Physics.

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